Coordinating microgrid procurement decisions with a dispatch strategy featuring a concentration gradient

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**HIGHLIGHTS**

- We combine a year-long hourly procurement strategy with minute-level dispatch.
- We linearize our model to increase tractability.
- We use a battery model derived from electrochemical principles.
- We include temperature and voltage transient effects via concentration gradients.
- Solutions from the minute-level model closely match load.

**ABSTRACT**

A mathematical model designs and operates a hybrid power system consisting of diesel generators, photovoltaic cells and battery storage to minimize fuel use at remote sites subject to meeting variable demand profiles, given the following constraints: power generated must meet demand in every time period; power generated by any technology cannot exceed its maximum rating; and best practices should be enforced to prolong the life of the technologies. We solve this optimization model in two phases: (i) we obtain the design and dispatch strategy for an hourly load profile, and (ii) we use the design strategy, derived in (i), as input to produce the optimal dispatch strategy at the minute level. Our contributions consist of: combining a year-long hourly optimization procurement strategy with a minute-level dispatch strategy, and using a high-fidelity battery model at the minute-level derived from electrochemical engineering principles that incorporate temperature and voltage transient effects. We solve both phases of the optimization problem to within 5% of optimality and demonstrate that solutions from the minute-level model more closely match the load, more closely capture battery and generator behavior, and provide fuel savings from a few percent to 30% over that provided by the hour-level model for the tested scenarios.

**1. Introduction**

A Forward Operating Base (FOB) is a secured military facility used to support tactical operations in foreign areas for several months to a few years. Scioletti et al. [1] propose an optimization model at a one-hour time fidelity over an annual horizon to determine the mix of equipment and the corresponding dispatch strategy at FOBs to reduce costs and fossil fuel consumption. By contrast, we present a two-phase model: Phase I, which we term \( \mathcal{P} \), and base off the work of Scioletti et al. [1], takes as input hourly energy demand, solar irradiance, temperature data and fuel cost and determines a design strategy, i.e., how many and what size generators, batteries, and photovoltaic (PV) cells to purchase to reliably power an off-grid site. The objective of our Phase I model minimizes diesel generator fuel consumption; constraints prevent over-cycling, enforce power limitations, and ensure that demand is met in each time period. Primarily, we use the hourly dispatch to inform procurement decisions; however, we cannot dispatch at the hourly level nor forecast demand for a year. Therefore, we construct a minute-level model to allow for near-real time dispatch.

Phase II, which we term \( \mathcal{M} \), uses the design strategy from \( \mathcal{P} \),

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minute-level energy demand, and temperature data to provide a minute-level dispatch strategy, i.e., how those technologies are operated, in each time period, to optimally meet the load for a 24-h horizon, i.e., 1440 time periods. The objective is largely the same as that used in the Phase I model with the exception of the procurement decisions that constitute a sunk cost, and the photovoltaic system that supplies as much power as the product of the anticipated solar panel output and the number of panels [2]. Other contrasting aspects follow: The battery system must start and end the day with an 80% state-of-charge while adhering to its limitations and inefficiencies. The generators may supply nameplate power rating conditional on restrictions regarding ramping times and minimum down times. The corresponding dispatch strategy schedules equipment usage and helps a microcontroller-based power management system make real-time dispatch decisions.

The remainder of the paper is organized as follows: Section 2 provides a review of relevant literature. Section 3 details first our hour-time-fidelity model, followed by our minute-time-fidelity model; regarding the latter, we explain the relevant detailed battery chemistry. Section 4 provides the mathematical formulation and its linearization of the optimization model at the minute-time-fidelity level; we provide a citation for the hour-level equivalent. We then give numerical results in Section 5 consisting of a description of the data, the benefits of using the minute-time-fidelity model over its hour-fidelity counterpart, and a parametric analysis based on varying some of the input values. Section 6 concludes.

2. Literature review

The literature on microgrid optimization is vast, and we provide a brief overview here. Many researchers investigate optimizing hybrid systems, both those that are grid-connected and those that are off-grid [3–8]; however, these authors do not employ mixed integer linear programs (MILPs), which produce a provably (near-)optimal solution. The following research all involves using heuristic, rather than exact, approaches to determine a solution: Abido [9] solves a multi-objective optimization model by a Pareto evolutionary algorithm based on fuzzy logic to develop a dispatch strategy. This approach shows promise in providing good, but not optimal, results to energy dispatch problems. Moghaddam [10] solves a multi-objective optimization model by minimizing fuel cost and emissions while adhering to power balance, generation, and transmission constraints with an adaptive modified particle swarm technique. Ashari [11] develops a dispatch strategy for a photovoltaic, diesel generator, and battery hybrid system for which a heuristic determines the time periods in which it is more advantageous to draw power from a battery than to run a generator at low power output, i.e., low efficiency. Park [12] uses tabu search, simulated annealing, and particle swarm optimization to solve non-smooth economic dispatch problems.

Research by [13–17] use MILPs to optimize distributed generation systems using exact approaches; however, none of these considers a minute-level time step, nor do even more computationally complex non-linear models that restrict the time horizon to 24 hours [18,19].

Batteries in hybrid systems are usually treated simplistically [16], whereas stand-alone battery systems are modeled in more detail. Barley [20] develops battery charging and discharging techniques for stand-alone power systems using a diesel generator, wind and/or solar photovoltaic, and battery systems. Achaibou [21] shows how to model voltage levels. Dufo-López [22] more accurately models the aging of a battery system in a solar photovoltaic and lead-acid battery system. Gao [23] models the complete behavior of a lithium-ion battery with thermal effects and response to transient power demand.

We present two different optimization models that contain batteries, varying in their level of detail. The first model, (\( \mathcal{H} \)), employs an hourly-fidelity, linearized model presented in [1], which expresses voltage of the battery as a linear function of the state-of-charge and employs a linear version of Peukert’s law [24]. Such models have been extensively used in the literature [25]. The battery model used the second optimization model, (\( M \)), captures minute-level battery operations in which the voltage transients along with temperature effects become critical. Here, we simplify the model based on an electrochemical principle presented by Guo et al. [26] by employing a polynomial approximation, as presented by Subramanian et al. [27]. The final form of the model can also be found in work by Ramachandran et al. [28].

Once a dispatch strategy has been established, a project management system controls the equipment in real time. Tavzunga [29] shows how a model-predictive controller yields better performance than an open-loop controller for a diesel generator, wind, solar photovoltaic, and battery system. McLarty [30] optimizes the dispatch of a multi-chiller cooling plant with cold-water thermal storage using a project management system for the UC Irvine microgrid. We construct an optimization model for a hybrid system whose solutions are usable in near-real time.

Our contributions in this paper include: (i) a mixed integer programming approach to developing an energy dispatch strategy for a microgrid using minute-level time fidelity with a concentration gradient for the battery and ramping effects for the generators; and (ii) the integration of this minute-level model with the decisions from an hourly model, and associated analysis between the compatibility of the two models.

3. Background

Our two models, (\( \mathcal{H} \)) and (\( M \)), work in tandem to determine a procurement strategy given a coarse dispatch strategy and then subsequently refine the dispatch strategy given the procurement strategy. We detail these models in this section, including the more precise characterization of the generator and battery behavior in the latter model. We give a flow chart of our methodology in Fig. 1.

3.1. Hour-level battery model

Phase I of our hybrid optimization model (\( \mathcal{H} \)) is a linearized model and considers parameters such as fuel cost, procurement costs of the power-producing technologies (i.e., generators, batteries, and photovoltaic cells), lifecycle cost of batteries and generators, and the electric efficiencies of power flow into and out of these technologies. In addition, the model incorporates energy demand, solar irradiance, and temperature effects at the site for each (hourly) time period during the horizon. The number and type of technologies to be procured, as well as the amount of fuel used, are variables within the optimization model. In addition to procurement decisions, the model also provides the dispatch of the hybrid system, which includes when each technology is turned on or off, for how long, and how much power it should produce within each hourly time period. This model is modified from that in Scioletti et al. [1] to include the effects of temperature on the performance of generators and batteries.

Specifically, the hourly battery model presented by Scioletti et al. [1] treats internal resistance (\( r_{ib} \)) and the rate capacity parameters (\( C_{ib} \)) as being independent of temperature. In this paper, we extend that model to include temperature effects in these parameters. By using Arrhenius-type temperature dependence (due to the fact that solid phase diffusion and reaction rate coefficients conform [31]), the parameters \( \beta_{ib} \) and \( \beta_{ib} \), expressed in Eqs. (1) and (2), are multiplied by internal resistance (\( r_{ib} \)) and rate capacity (\( C_{ib} \)) to give rise to temperature-dependent parameters (\( r_{ib}^{T} \) and \( C_{ib}^{T} \)). The battery temperature is assumed to be the same as the ambient temperature and, therefore, known a priori, allowing us to treat the temperature dependence as a parameter, rather than a variable. While internal resistance changes the voltage of the battery up and down during charging and discharging, respectively, the rate capacity parameter places an upper bound on allowable current in a given time period. Table 1 provides a contrast between the
3.2. Minute-level battery model

In order to develop dispatch strategies at the minute level, the battery model must capture the transient effects in voltage behavior. In a lithium-ion battery (LiB), various transport phenomena (e.g., diffusion of lithium in solid particles and diffusion and migration of lithium ions in an electrolyte) have associated transient effects. The concentration gradient is the gradient of lithium in solid particles at the anode and cathode. At the minute-level, the transient effects due to the solid phase diffusion of lithium at the anode and cathode have a larger effect on voltage than the transient effects due to the diffusion of lithium ions in an electrolyte.

During charging (discharging) of a LiB, lithium moves from the cathode (anode) side to the anode (cathode) side. Fig. 2 (left) shows the three layers: anode, separator, and cathode, where the anode and cathode are porous structures consisting of micron-sized particles typical in a lithium-ion battery. This single-particle diagram focuses on the diffusion of lithium in solid particles; the red curves illustrate the concentration profile. An electrolyte (i.e., an organic solvent with lithium salt), which is present in the pores of the layers, facilitates the movement of lithium between the anode and cathode. A rigorous electrochemical model [32] for a LiB may include: transport of lithium ions in the electrolyte phase, diffusion of lithium in the solid phase, Butler-Volmer reaction kinetics, and the transport of electrons. In most

![Diagram](image-url)
commercial LiBs, the diffusion of lithium in the solid phase becomes the dominant source of voltage transience, resulting in many simplified electrochemical models, including ours, focusing on the diffusion in a single particle at the anode and cathode, ignoring other transport effects [26].

The model presented by Guo et al. [26] (also known as a single-particle model) consists of two partial differential equations (Fick’s second law) which describes diffusion of lithium in solid spherical particles at the anode and cathode. The voltage of a battery relates to the concentration of lithium at the surface of these solid particles as chemical reactions occur at their surface. The single-particle diagram can be simplified by approximating the concentration profiles in the solid particles. One common approach in calculating lithium concentration is to use a polynomial approximation for the concentration profiles to reduce the severity of the nonlinearities.

We use a quartic approximation to capture the concentration profile of lithium in solid particles, as presented by Subramanian et al. [27]. The solution of Fick’s law of diffusion (indicated by the red curves in Fig. 2 for each of the solid particles of the anode and cathode) is assumed to take a fourth-order polynomial form. The quartic approximation leads to three equations in each particle related to three variables: average mole fraction, surface mole fraction, and volume-averaged mole fraction flux. The voltage of a battery is related to the surface mole fraction of lithium at the anode and cathode. The time scale for the diffusion process is on the order of minutes (radius/particle²/diffusivity). In other words, it requires this amount of time to fully develop the concentration profile in a solid particle as the transient step. Three equations for each particle (each at the anode and cathode) and the equation for voltage give rise to seven total battery model equations. The differential equations can be converted to difference equations with one-minute time steps. The following section introduces the mathematics of the concentration gradient.

3.3. Battery chemistry modeling

We first present the parameters based on battery chemistry, structure details, material properties, and transport properties, followed by variables that relate to concentration gradients at the anode and cathode. The remainder of this section explains the mathematical model relating concentration and voltage for a specific battery. We generalize this in the minute-level optimization model in Section 4.1. Note that the superscripts p and n refer to the cathode and anode, respectively.

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<th>Battery-model parameters</th>
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Battery-model variables

$C^p(t)$ | Average mole fraction of lithium in the cathode at time t [unitless] |
| $C^n(t)$ | Average mole fraction of lithium in the anode at time t [unitless] |
| $Q^p(t)$ | Volume averaged mole fraction flux of lithium in the cathode at time t [unitless] |
| $Q^n(t)$ | Volume averaged mole fraction flux of lithium in the anode at time t [unitless] |
| $S^p(t)$ | Surface mole fraction of lithium in the cathode at time t [unitless] |
| $S^n(t)$ | Surface mole fraction of lithium in the anode at time t [unitless] |

Differential Eqs. (3) and (4) express the material balance of lithium at the positive and negative electrodes (i.e., at the cathode and anode, respectively). When the battery is being charged ($I^+ > 0$ and $I^- = 0$), the average mole fraction of lithium at the cathode, $C^p(t)$, decreases, whereas the average mole fraction of lithium at the anode, $C^n(t)$, increases [27]:

$$\frac{dC^p(t)}{dt} = -k^{p3}(I^+(t) - I^-(t))$$  \hspace{1cm} (3)

$$\frac{dC^n(t)}{dt} = k^{n3}(I^+(t) - I^-(t))$$  \hspace{1cm} (4)

The parameters used in the equations above are related to battery structure and geometry ($\alpha^c$, $\rho_{p/n}$, $\epsilon_{p/n}$), chemistry ($\rho_{p/n}$) and configuration (q); the parameters $k^{p3}$ and $k^{n3}$, which relate the charge transfer during that time period to average mole fractions, are given by the following expressions:

$$k^{p3} = \frac{1}{F\cdot(1-\epsilon^p)\cdot l^p\cdot c^p\cdot \alpha^c\cdot q}$$  \hspace{1cm} (5)

$$k^{n3} = \frac{1}{F\cdot(1-\epsilon^n)\cdot l^n\cdot c^n\cdot \alpha^n\cdot q}$$  \hspace{1cm} (6)

Eqs. (7) and (10) show how the volume-averaged mole fraction flux for the cathode and anode ($Q^p(t)$ and $Q^n(t)$, respectively) change with current and time. These equations incorporate the transient effects in voltage behavior as follows:

$$\frac{dQ^p(t)}{dt} = -k^{p4}\cdot 30d^p\cdot Q^n(t) - k^{p5}\cdot (I^+(t) - I^-(t))$$  \hspace{1cm} (7)

where:

$$k^{p4} = \frac{30d^p}{F^{3/2}}$$  \hspace{1cm} (8)

$$k^{p5} = \frac{15}{2\cdot F\cdot(1-\epsilon^p)\cdot l^p\cdot c^p\cdot \alpha^c\cdot q}$$  \hspace{1cm} (9)

In Eq. (10), $k^{n4}$ and $k^{n5}$ are given by expressions similar to (8) and (9), changing cathode to anode.

$$\frac{dQ^n(t)}{dt} = -k^{n4}\cdot 30d^n\cdot Q^p(t) + k^{n5}\cdot (I^+(t) - I^-(t))$$  \hspace{1cm} (10)

The diffusion coefficient ($d^p$) of lithium in its solid phase appears in Eq. (13), and is also given in the concentration gradient parameter list.
The presence of numbers such as 30 and 15 is due to the integration of quartics (see Subramanian et al. [27]).

Based on the average mole fraction and the volume-averaged flux, the surface mole fraction at the anode and cathode are given by Eqs. (11) and (12), respectively.

\[ S^p(t) = \frac{R}{35}Q^p(t) - \frac{k^{p6}(I^p(t)-I^6(t))}{\partial \phi^p} \]  

\[ S^c(t) = \frac{R}{35}Q^c(t) + \frac{k^{c6}(I^c(t)-I^6(t))}{\partial \phi^c} \]  

where the parameter \( k^{p6} \) is expressed as follows:

\[ k^{p6} = \frac{r^2}{105F(1-e^\tau)} \]  

The surface mole fraction of the solid particles determines the voltage of the battery in the minute model, as opposed to using the average mole fraction to determine the voltage in the hour model. Eqs. (11) and (12) show that the average mole fraction differs from the surface mole fraction. By introducing the volume-averaged mole fraction flux and surface mole fraction, transient effects appear in the battery voltage. The voltage prediction in the hourly model depends on the state-of-charge (SoC), which is equivalent to using only the average mole fraction and ignoring the concentration gradients in the particles.

The battery voltage, which depends on the surface mole fractions \( S^p(t), S^c(t) \) and the voltage correction due to internal resistances (e.g., transport, reactions, contact, wiring), can be expressed with the help of the equations given above. These internal resistances can be highly nonlinear; here, we use the linearized resistances. The LiB voltage is a nonlinear function of the surface mole fraction of lithium at the anode and cathode. We linearize the open circuit voltage by creating a linear approximation around the 50% state-of-charge point. Fig. 3 shows the experimentally obtained open circuit voltage (blue dashed curve) as well as the linearized voltage (solid red line) of a Panasonic 18650B 3.4 Ah cell versus the capacity (state-of-charge).

### 4. Minute-level optimization model

The model new to this research determines a dispatch strategy given a procurement strategy from \( \mathcal{P} \), an established model [1]. The dispatch variables represent the levels at which the purchased technologies operate. The objective of our model is to maximize the optimal power output for each device for one day at a minute-level time fidelity subject to meeting power demand and limitations of the power generation technologies. The concentration gradient effects help to more accurately model the performance of the batteries; we include ramping effects of the generators as well.

![Open circuit voltage vs. capacity](image)

**Fig. 3.** The open circuit voltage (blue dashed curve) of a battery versus capacity (state-of-charge), obtained by discharging a fully charged battery very slowly, and linearized open circuit voltage (red solid line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

### 4.1. Mathematical formulation

We now present the mathematical formulation of our problem \( \mathcal{M} \). In general, we use lower-case letters for parameters and upper-case letters for variables except for established constants. We also use lower-case letters for indices and upper-case script letters for sets. Superscripts and accents distinguish between parameters and variables that utilize the same base letter, while subscripts identify elements of a set. Some parameters and variables are only defined for certain set elements, which are listed in each definition. Plus signs (+) signify power entering a technology, while minus signs (−) indicate power leaving. The units of each parameter and variable are provided in brackets after its definition. We use the term “twins” to denote a tuple or a multiple of a certain technology type.

### Sets

- \( t \in \mathcal{T} \): Set of all time periods [minutes], 1,...,|\( \mathcal{T} \)|
- \( j \in \mathcal{J} \): Set of all battery and generator technologies
- \( g \in \mathcal{G} \subset \mathcal{J} \): Set of all generator technologies
- \( b \in \mathcal{B} \subset \mathcal{J} \): Set of all battery technologies
- \( s \in \mathcal{S} \): Set of all PV panel types
- \( k \in \mathcal{J}_g \): Set of twins of technology \( j \), given specifically by size, type, and manufacturer
- \( k \in \mathcal{B}_g \): Set of all battery twins of type \( b \)

### Timing parameters

- \( \tau \): Length of one time period [hours]
- \( \nu \): Ratio of base operation duration to time horizon length [fraction]

### Optimization model penalty parameters

- \( \bar{C}_j \): Cost of procuring one twin of technology type \( j \) [$/twin$]
- \( c_s \): Cost of procuring one panel of technology type \( s \) [$/panel$]
- \( g/ \): Fuel cost penalty in time period \( t \) [$/gal$]
- \( \bar{C}_j \): Cycle cost penalty for technology type \( j \) [$/cyle (minutes, cycles)]
- \( \bar{W}_j \): Weight of one twin of technology type \( j \) [kg/twin]
- \( w_s \): Weight of procuring one PV system of type \( s \) [kg/ system]
- \( w/ \): Weight of fuel [kg/gal]
- \( \omega \): Transportation cost per unit of equipment or of fuel weight [$/kg$]
- \( \mu_{\ell} \): Relative objective weight for cost minimization [fraction]
- \( \mu_{\ell} \): Relative objective weight for life cycle minimization [fraction]
- \( \mu_{\ell} \): Relative objective weight for weight minimization [fraction]

### Power system parameters

- \( d/ \): Average power demand in time period \( t \) [W]
- \( \mu \): Overage load coefficient [fraction]
- \( k/ \): Spinning reserve required relative to PV power [fraction]
- \( \mu \): Spinning reserve coefficient [fraction]

### Technology parameters

- \( \ell_j \): Maximum lifetime of technology type \( j \) [generator minutes, battery cycles]
- \( \eta_j^+, \eta_j^- \): Electric efficiency of power flow into and out of technology type \( j \), respectively [fraction]
**Generator parameters**

- $a_g^j$, $b_g^j$, $c_g^j$: Fuel consumption coefficients for generator $g$ in [Mgal, gal, gal]
- $\epsilon_{g}^\text{up}$, $\epsilon_{g}^\text{down}$: Ramp up and down time, respectively, for generator $g$ in [fraction/minutes]
- $\kappa_{g}^\text{up}$, $\kappa_{g}^\text{down}$: Minimum up and down time, respectively, for generator $g$ in [minutes]
- $k_{g}^{ab}$: Electric derating factor of generator $g$ for altitude [fraction]
- $k_{g}^{\text{comp}}$: Electric derating factor of generator $g$ in time $t$ for temperature [fraction]

**Battery parameters**

- $a_b^+$, $a_b^-$: Bi-directional converter slope-intercept parameter for battery $b$ in [W]
- $\beta_b^+$, $\beta_b^-$: Bi-directional converter slope parameter for battery $b$ [unitless]
- $a_b^+, b_b^+$: Battery $b$ linear voltage model slope and intercept coefficients, respectively [V]
- $a_b^\text{soc}$, $d_b^\text{soc}$: Battery $b$ linear lifetime model slope and intercept coefficients, respectively [unitless]
- $b_b^0$: Battery $b$ state-of-charge used in initial condition constraints [fraction]
- $c_b^\text{ref}$: Battery $b$ manufacturer-specified capacity [Ah]
- $c_b^+, c_b^-$: Battery $b$ charge and discharge capacity rate coefficients, respectively [hours]
- $\theta_b$: Battery $b$ internal resistance [Ohm]
- $\delta_b$: Typical current expected from battery $b$ for both charge and discharge activities [A]
- $\sigma_b$, $\xi_b$: Battery $b$ state-of-charge maximum and minimum operational bounds, respectively [fraction]
- $i_b^+, i_b^-$: Battery $b$ charge current upper and lower bound, respectively [A]

- $i_b^+ = \frac{c_b^\text{ref}}{c_b^0} + \delta_b \quad \forall b \in B$
- $i_b^- = 0 \quad \forall b \in B$
- $i_b^+, i_b^-$: Battery $b$ discharge current upper and lower bound, respectively [A]

- $i_b^+ = 2\delta_b \quad \forall b \in B$
- $i_b^- = 0 \quad \forall b \in B$
- $v_b^\text{nom}$: Nominal voltage for battery $b$ [V]

- $v_b^\text{nom} = a_b^0 + b_b^0 - i_b^- i_b^+ r_b^\text{nom} \quad \forall b \in B$
- $v_b^+ = \text{Electrolyte resistance for battery } b$ [Ohm]
- $r_b^-$: Contact resistance for battery $b$ [Ohm]
- $b_b^+$, $b_b^\text{ss}$: Constant for the solid state potential in the positive and negative terminal of battery $b$, respectively [V]
- $c_b^\text{ref}$: Constant for the solid state potential of battery $b$ [Ohm]
- $k_{b}^{ss}$, $k_{b}^{\text{ref}}$: Battery $b$ constant for average mole fraction equation for charging and discharging from Eqs. (5) and (6), respectively [$\frac{1}{A}$]

**PV parameters**

- $\gamma_s$: Power output of technology type $s$ in time period $t$ in [Wpt]
- $\alpha_s$: Number of PV panels procured of technology type $s$ [panels]

**Continuous variables**

- $L_k$: Number of expended life cycles for technology type $j$, twin $k$ [generator minutes, battery cycles]
- $P_{\text{fkt}}^\text{in}, P_{\text{fkt}}^\text{out}$: Aggregate power into and out of technology type $j$, twin $k$ in time period $t$, respectively [W]
- $P_s^\text{PV}$: Aggregate power out of PV technology type $s$ in time period $t$ [W]

**Generator variables**

- $\bar{F}_j$: Amount of fuel used in time period $t$ [gal]

**Battery variables**

- $B_b^\text{soc}$: State-of-charge of battery type $b$, twin $k$ at the end of time period $t$ [fraction]
- $B_b^\text{ss}$: Maximum capacity available of battery type $b$, twin $k$ in time period $t$ [A]
- $I_{b,\text{cur}}, I_{b,\text{ref}}$: Battery $b$, twin $k$ current for charge and discharge, respectively, in time period $t$ [A]
- $V_b^\text{soc}$: Battery $b$, twin $k$ voltage in time period $t$ [V]
- $Q_{b,\text{cur}}^\text{c}, Q_{b,\text{ref}}^\text{c}$: Average mole fraction of lithium in the cathode and anode of battery type $b$, twin $k$ in time period $t$, respectively [unitless]
Surface mole fraction of lithium in the cathode and anode of battery type b, twin k in time period t, respectively [unitless]

Binary and integer variables

**Generator variables**

- $G^{\text{init}}_b$ if technology type g, twin k is operating in time period t, 0 otherwise
- $G^{\text{stop}}_b$ if technology type g, twin k is ramping up in time period t, 0 otherwise
- $G^{\text{stop}}_b$ if technology type g, twin k is ramping down in time period t, 0 otherwise

**Battery variables**

- $B^{\text{c}}_b$ if battery type b, twin k is charging in time period t, 0 otherwise
- $B^{\text{d}}_b$ if battery type b, twin k is discharging in time period t, 0 otherwise

**Problem (M)**

(see Section 4.2.1 Objective Function)

minimize $\mu \left( \sum_{i \in S} \sum_{k \in \mathcal{K}} \mu_i \sum_{j \in \mathcal{J}} \delta_{j, t} T_j + \mu \sum_{i \in S} \sum_{k \in \mathcal{K}} \delta_{i, k} \right) + \mu \alpha \sum_{i \in S} \sum_{k \in \mathcal{K}} \delta_{i, k} \left( \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \tilde{W}_j \right)$

subject to

(see Section 4.2.2 System Operations)

(14a) $\sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{K}} n_i \sum_{s \in \mathcal{S}} \left( \frac{P_{i, k}^{\text{in}} - n_i}{1 + P_{i, k}^{\text{in}}} - \sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{K}} \left( \frac{P_{i, k}^{\text{in}} + n_i}{P_{i, k}^{\text{out}}} + \delta_{i, k} B_{i, k}^{\text{init}} \right) \right)$

(15a) $\sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{K}} B_{i, k}^{\text{out}} \left( \frac{n_i^2}{n_i^2 - P_{i, k}^{\text{in}}} - (1 + P_{i, k}^{\text{in}}) \right) + \sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{C}} \left( \nu_i \kappa_i k^{\text{mp}} G_{i, k}^{\text{init}} - P_{i, k}^{\text{out}} \right) \geq 0$

(15b) $P_{i, k}^{\text{in}} \leq B_{i, k}^{\text{out}} \leq P_{i, k}^{\text{max}} \forall b \in \mathcal{B}, k \in \mathcal{K}, t \in \mathcal{T}$

(15c) $B_{i, k}^{\text{init}} \leq \frac{P_{i, k}^{\text{in}} - n_i}{1 + P_{i, k}^{\text{in}}} \frac{P_{i, k}^{\text{out}}}{P_{i, k}^{\text{out}}} \sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{C}} \left( \nu_i \kappa_i k^{\text{mp}} G_{i, k}^{\text{init}} - P_{i, k}^{\text{out}} \right) \geq 0$

(15d) $P_{i, k}^{\text{in}} \leq B_{i, k}^{\text{out}} \leq P_{i, k}^{\text{max}} \forall b \in \mathcal{B}, k \in \mathcal{K}, t \in \mathcal{T}$

(see Section 4.2.3 Generator Operations)

(16a) $P_{i, k}^{\text{init}} \leq P_{i, k}^{\text{out}} \leq P_{i, k}^{\text{max}} \forall g \in \mathcal{G}, k \in \hat{G}_b, t \in \mathcal{T}$

(16b) $T_j \geq \sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{K}} \left( a_j P_{i, k}^{\text{in}} + b_j P_{i, k}^{\text{out}} + c_j G_{i, k}^{\text{init}} \right) \forall j \in \mathcal{J}, t \in \mathcal{T}$

(see Section 4.2.4 Generator Ramp-Up and Ramp-Down Time)

(17a) $P_{i, k}^{\text{out}} \geq P_{i, k}^{\text{in}} \kappa_i k^{\text{mp}} \sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{C}} \left( \nu_i \kappa_i k^{\text{mp}} G_{i, k}^{\text{init}} - P_{i, k}^{\text{out}} \right) \forall g \in \mathcal{G}, k \in \hat{G}_b, t \in \mathcal{T}$

(17b) $P_{i, k}^{\text{in}} \geq P_{i, k}^{\text{out}} \kappa_i k^{\text{mp}} \sum_{i \in \mathcal{S}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{C}} \left( \nu_i \kappa_i k^{\text{mp}} G_{i, k}^{\text{init}} - P_{i, k}^{\text{out}} \right) \forall g \in \mathcal{G}, k \in \hat{G}_b, t \in \mathcal{T}$

(see Section 4.2.5 Generator Minimum-Up and -Down Time)
4.2. Detailed discussion of formulation

4.2.1. Objective function

The objective function (14) minimizes the sum of three terms: (i) the cost associated with procuring various technologies and generator fuel; (ii) an arbitrarily weighted measure of the life cycles used by each technology over the total length of operation; and (iii) a weighted measure of the transportation cost associated with the various technologies and fuel.

4.2.2. System operations

Constraint (15a) ensures that the minute demand for power is met. The first term represents the power being discharged from the batteries, accounting for inverter losses; the second term represents the power to charge the batteries, accounting for rectifier losses; the third term represents the power from the generators, accounting for power system losses; and the forth term reflects the contributions of PV power. The right-hand side is the product of the forecasted demand for the time period and an average load factor. Constraint (15b) enforces “spinning reserve” via the use of a dispatchable technology (generator or battery) capable of meeting a fraction of the power provided by PV. Constraint (15c) bounds the capacity of a battery to a minimum and maximum value. Constraint (15d) ensures the battery capacity is less than the maximum level. Constraint (16b) determines the amount of fuel used by a generator during time period t.

4.2.4. Generator ramp-up and ramp-down time

Constraint (17a) limits the increase in the power output of a given generator from the previous time period (t-1) to the current time period (t). Constraint (17b) limits the decrease in the power output of a given generator from the previous time period (t-1) to the current time period (t).

4.2.5. Generator minimum-up and -down time

Constraints (18a) and (18b) determine the time period in which a given generator starts and stops, respectively. Constraints (18c) and (18d) maintain that the given generator continues to run or stay stopped until at least $t_{sp}^\gamma$ or $t_{down}^\gamma$ has passed, respectively.

Constraints (18e) and (18f) break symmetry by numerically prioritizing the use of procured generator twins within a technology set given minimum up and minimum down time constraints.

4.2.6. PV operations

We limit the PV output power of technology type $s$ in time period $t$ per panel to $P_{fi}^\gamma$, in constraint (19a). The anticipated solar panel output results from a PVWatts simulation run a priori, which accounts for performance characteristics such as location, panel efficiency, tilt, and angle.

4.2.7. Battery storage operations

Constraints (20a) and (20b) represent the nonlinear relationship between voltage, current, and the power associated with charging and discharging the battery, respectively. Constraint (20c) provides bounds for charging current.

Constraint (20d) updates the battery SOC, which is a function of its previous SOC and the discharge and charge currents. The battery must start and end the day with the same SOC.

Constraint (20e) bounds the SOC of a battery to a minimum and maximum level. Constraint (20f) ensures that the batteries operate in droop, rather than individually. Constraints (20g) and (20h) determine the change in average mole fraction at the anode and cathode particles derived from Eqs. (3) and (4), respectively. Constraints (20i) and (20j) determine the volume average mole fraction flux at the anode and cathode particles derived from Eqs. (7) and (10), respectively. Constraints (20k) and (20l) determine the surface mole fraction at the anode and cathode particles derived from Eqs. (11) and (12), respectively.

Constraint (20m) and (20n) set upper and lower bounds for the surface mole fraction at the anode and cathode particles, respectively. Constraint (20o) models the battery voltage as a function of surface mole fraction and the direction of current flow. Constraints (20p) and (20q) bound the net power flow of each battery per time period, while constraints (20r), (20s), and (20t) similarly constrain current flow. Constraint (20u) prevents simultaneous charge and discharge operations.

4.2.8. Lifecycle

Constraint (21a) counts the number of operational minutes of a generator. Constraint (21b) counts life cycles for batteries. A battery’s lifecycle is a function of both the amount of the charge and discharge currents as well as the SOC level at which the charge or discharge occurs. Because the lifecycle constraint considers both charge and discharge i.e., two opposite-direction operations, which together we refer to as a round-trip, we divide by two. Constraint (21c) limits technology lifetime.

4.2.9. Non-negativity and Integrality

Finally, constraints (22a), (22b), (22c), (22d), (22e) ensure that the appropriate variables in our formulation assume continuous, non-negative values. Constraint (22f) allows battery flux to be unrestricted.
Constraints (22g) and (22h) enforce integer and binary restrictions, as appropriate.

4.3. Linearization

(Model) is nonlinear in that there is one quadratic term (see constraint (16b)), and bilinear terms within constraints (20a), (20b), and (21b). Models with similar nonlinearities solved using instances with sizes on par with ours report long run times [1]. Therefore, to increase tractability of the corresponding model instances, we present (Model), a linearization of (Model), which also corresponds to an under-estimation of the original problem.

4.3.1. Exact linearization

We can approximate a quadratic function by using piecewise linear functions; however, in our case, the data provided by the manufacturers corresponds to a line, so we set \(a_i\) equal to 0, thereby eliminating the quadratic term. The bilinear terms assume the product of two continuous variables and we use a convex under-estimation technique. We do not explicitly present the constraints for the case in which \(a_i\) replaces \(b_i\) (which occurs when \(a_i > b_i\)).

Substituting the voltage constraint (20a) directly into the power constraints (20a) and (20b), we obtain:

\[
P_{\text{eff}}^b = (a_i^b-b_i^b S_{\text{ref}}^b+b_i^b S_{\text{ref}}^b+(c_i^b+c_i^b+r_i^b)(I_{\text{soc}}-I_{\text{soc}}^b))I_{\text{soc}}^b
\]

\(\forall b \in B, \ k \in \hat{B}_n, \ t \in \tau\)  \hspace{1cm} (23)

\[
P_{\text{eff}}^b = (a_i^b-b_i^b S_{\text{ref}}^b+b_i^b S_{\text{ref}}^b+(c_i^b+c_i^b+r_i^b)(I_{\text{soc}}-I_{\text{soc}}^b))I_{\text{soc}}^b
\]

\(\forall b \in B, \ k \in \hat{B}_n, \ t \in \tau\)  \hspace{1cm} (24)

We can simplify Eqs. (23) and (24) by distributing the respective current variable and replacing the current on the last term with \(I_{\text{soc}}^b\).

\[
P_{\text{eff}}^b = a_i^b I_{\text{soc}}^b-b_i^b S_{\text{ref}}^b I_{\text{soc}}^b+b_i^b S_{\text{ref}}^b I_{\text{soc}}^b+(c_i^b+c_i^b+r_i^b)I_{\text{soc}}^b I_{\text{soc}}^b
\]

\(\forall b \in B, \ k \in \hat{B}_n, \ t \in \tau\)  \hspace{1cm} (25)

\[
P_{\text{eff}}^b = a_i^b I_{\text{soc}}^b-b_i^b S_{\text{ref}}^b I_{\text{soc}}^b+b_i^b S_{\text{ref}}^b I_{\text{soc}}^b-(c_i^b+c_i^b+r_i^b)I_{\text{soc}}^b I_{\text{soc}}^b
\]

\(\forall b \in B, \ k \in \hat{B}_n, \ t \in \tau\)  \hspace{1cm} (26)

We distribute the terms on the righthand side of the lifecycle constraint (21b) to identify bilinear terms consisting of SOC and current:

\[
I_{\text{soc}}^b \geq \sum_{i \geq 2} \left( \frac{a_i^b I_{\text{soc}}^b-d_i^b I_{\text{soc}}^b-d_i^b I_{\text{soc}}^b B_{\text{soc},1}^b I_{\text{soc}}^b}{2c_i^b} \right) \forall b \in B, \ k \in \hat{B}_n
\]

\hspace{1cm} (27)

**Auxiliary variables**

Eqs. (25) and (26), and (27) contain bi-linear terms, for which we define nonnegative continuous variables:

\[
Z_{b,k,t}, \ Z_{b,k,t}^{-}\quad \text{Battery b, twin k linear approximation variable representing the product of state-of-charge and current for charge and discharge, respectively, in time period t [A]}
\]

\[
Z_{b,k,t}^{1}, \ Z_{b,k,t}^{1-}\quad \text{Battery b, twin k linear approximation variable representing the product of cathode surface mole fraction and current for charge and discharge, respectively, in time period t [A]}
\]

\[
Z_{b,k,t}^{2}, \ Z_{b,k,t}^{2-}\quad \text{Battery b, twin k linear approximation variable representing the product of anode surface mole fraction and current for charge and discharge, respectively, in time period t [A]}
\]

\[
Z_{b,k,t}^{+}, \ Z_{b,k,t}^{+}\quad \forall b \in B, \ k \in \hat{B}_n, \ t \in \tau
\]

\hspace{1cm} (28)

\[
Z_{b,k,t}^{-} = B_{\text{soc},1}^b I_{\text{soc}}^b \forall b \in B, \ k \in \hat{B}_n, \ t \in \tau; \ t > 1
\]

\hspace{1cm} (29)

\[
Z_{b,k,t}^{+} = B_{\text{soc},1}^b I_{\text{soc}}^b \forall b \in B, \ k \in \hat{B}_n, \ t \in \tau; \ t > 1
\]

\hspace{1cm} (30)
We then substitute these variables directly into (25), (26), and (27):

\[ P_{\text{bat}} = a_k^\text{i} I_{\text{bat}}^k - b_k^\text{i} Z_{\text{bat}}^k + b_k^{\text{soc}} Z_{\text{bat}}^{\text{soc}} + (c_k^\text{i} + c_k^{\text{soc}} + d_k^\text{i}) I_{\text{bat}}^k I_{\text{bat}}^k \quad \forall b \in B, k \in B_{\text{bat}}, t \in T \]  

(35)

\[ P_{\text{bat}} = a_k^\text{i} I_{\text{bat}}^k - b_k^\text{i} Z_{\text{bat}}^k + b_k^{\text{soc}} Z_{\text{bat}}^{\text{soc}} - (c_k^\text{i} + c_k^{\text{soc}} + d_k^\text{i}) I_{\text{bat}}^k I_{\text{bat}}^k \quad \forall b \in B, k \in B_{\text{bat}}, t \in T \]  

(36)

\[ L_{\text{bat}} \geq \tau \sum_{i \geq 1} \left( \frac{a_k^\text{soc} I_{\text{bat}}^k - d_k^\text{soc} Z_{\text{bat}}^k + a_k^\text{soc} I_{\text{bat}}^k - d_k^\text{soc} Z_{\text{bat}}^k}{2 \eta^\text{soc}} \right) \quad \forall b \in B, k \in B_{\text{bat}} \]  

(37)

Constraint (21b) presents a symmetric function that penalizes both charge and discharge operations equally as a fraction of capacity. Given our definition of SOC per constraint (20d), which implies that the battery needs to charge in order to discharge, we can simplify constraint (37) by multiplying it by two, which cancels the 2 in the denominator, and removing either the charge or discharge variables. We choose to remove the charge variables because approximations for \( Z_{\text{bat}}^k \) have been shown to be more accurate [16].

\[ L_{\text{bat}} \geq \tau \sum_{i \geq 1} \left( \frac{a_k^\text{soc} I_{\text{bat}}^k - d_k^\text{soc} Z_{\text{bat}}^k}{2 \eta^\text{soc}} \right) \quad \forall b \in B, k \in B_{\text{bat}} \]  

(38)

For all constraints involving bilinear terms, we invoke an approximation to eliminate the nonlinearities associated with \( Z_{\text{bat}}^k \), \( Z_{\text{bat}}^{\text{soc}} \), \( Z_{\text{bat}}^{\text{soc}} \), \( Z_{\text{bat}}^{\text{soc}} \), and \( Z_{\text{bat}}^{\text{soc}} \). This linearization in constraints (39a)–(39f), which replace constraints (29) and (34) in our reformulation.

\[ Z_{\text{bat}}^k \leq b_k^{\text{i}} E_{\text{soc}}^k I_{\text{bat}}^k + b_k^{\text{soc}} E_{\text{soc}}^{\text{soc}} - b_k^{\text{i}} I_{\text{bat}}^k U^{\text{i}} \quad \forall b \in B, k \in B_{\text{bat}}, t \in T : t > 1 \]  

(39a)

\[ Z_{\text{bat}}^k \leq b_k^{\text{i}} E_{\text{soc}}^k I_{\text{bat}}^k + b_k^{\text{soc}} E_{\text{soc}}^{\text{soc}} - b_k^{\text{i}} I_{\text{bat}}^k U^{\text{i}} \quad \forall b \in B, k \in B_{\text{bat}}, t \in T : t > 1 \]  

(39b)

\[ Z_{\text{bat}}^k \leq b_k^{\text{i}} E_{\text{soc}}^k I_{\text{bat}}^k + b_k^{\text{soc}} E_{\text{soc}}^{\text{soc}} - b_k^{\text{i}} I_{\text{bat}}^k U^{\text{i}} \quad \forall b \in B, k \in B_{\text{bat}}, t \in T : t > 1 \]  

(39c)

\[ Z_{\text{bat}}^k \leq b_k^{\text{i}} E_{\text{soc}}^k I_{\text{bat}}^k + b_k^{\text{soc}} E_{\text{soc}}^{\text{soc}} - b_k^{\text{i}} I_{\text{bat}}^k U^{\text{i}} \quad \forall b \in B, k \in B_{\text{bat}}, t \in T : t > 1 \]  

(39d)

\[ Z_{\text{bat}}^k \leq b_k^{\text{i}} E_{\text{soc}}^k I_{\text{bat}}^k + b_k^{\text{soc}} E_{\text{soc}}^{\text{soc}} - b_k^{\text{i}} I_{\text{bat}}^k U^{\text{i}} \quad \forall b \in B, k \in B_{\text{bat}}, t \in T : t > 1 \]  

(39e)

\[ Z_{\text{bat}}^k \leq b_k^{\text{i}} E_{\text{soc}}^k I_{\text{bat}}^k + b_k^{\text{soc}} E_{\text{soc}}^{\text{soc}} - b_k^{\text{i}} I_{\text{bat}}^k U^{\text{i}} \quad \forall b \in B, k \in B_{\text{bat}}, t \in T : t > 1 \]  

(39f)

5. Data and results

The linear optimization model uses the AMPL (A Modeling Language for Mathematical Programming) modeling language [35] and is solved with the CPLEX Solver, Version 12.6.0.1 [36] on a Dual-Quad core CPUs at 2.83 GHz with 16 GB RAM and 160 GB HDD under a Ubuntu 14.04 operating environment.

5.1. Data

We take the generator parameters from Table 2 of [1]. Additional (i.e., battery) parameters for the hourly model (\( H \)) are given in Table 2 (in this paper). Table 3 shows the measured values, obtained from a lithium-ion battery tested at Georgia Institute of Technology (Atlanta, Georgia, USA); we calculate \( k_b^i, k_b^{soc}, k_b^i, k_b^{soc}, k_b^i, k_b^{soc}, k_b^i, k_b^{soc}, k_b^i, \) and \( k_b^{soc} \) using Eqs. (3)–(13). Additional values for \( M \) are given in Table 4.

The Base Camp Integration Laboratory (BCIL) is located at F.t. Devens in Massachusetts and, inter alia, serves as a facility to test equipment used by soldiers during deployment. We received energy demand data from tests conducted at this facility, based on a simulated FOB containing buildings indicative of what the military uses, with
metered power demand information and weather data. Our demand data was collected at the minute-level for the 10 working days between June 6, 2016 and June 17, 2016 during Sustainability Logistics Basing - Science Technology Objective Demonstration activities at the BCIL. This demonstration is led by the Research, Development and Engineering Command, and is managed by the Natick Soldier Research, Development and Engineering Center. We use the data, as such in (M), and average the observations to obtain hour-level values for load suitable for (H). Our results focus on two days from the 10-day dataset: June 14 and 17, corresponding to Day 7 and Day 10, respectively. The other days exhibit similar results.

5.2. Model results and parametric study

The Phase I optimization model (H), solved for the 10-day time horizon, provides a procurement strategy of a 250 kW hours battery, 15 kW generator and 75 solar panels. This dispatch strategy is then fixed for both runs we perform with (M). Whereas (H) requires about 76 s to solve to a 5% gap for 240 time periods (or a 10-day horizon), instances of (M) containing 1,440 time periods (equating to a day-long time horizon) required about 15 minutes to solve. (H) provides a crude (hourly) dispatch without the detail of the concentration gradient for the batteries and ramping characteristics of the generators which is nonetheless helpful for informing the procurement strategy. Typically, given more extensive data, one would run (H) for an annual time horizon (see, e.g., [37]), precluding the inclusion of detailed, minute-level behavior that we are able to incorporate into (H). In order to mitigate the lack of clairvoyance in the minute model, we place extra constraints that the battery must begin (and, therefore, end) each day with a state-of-charge of 80%. As discussed below, this serves to promote use of the battery each day. While there are similarities between the dispatch strategies produced by (H) and (M), there are also clear advantages afforded by the more detailed model. Specifically, Figs. 5 and 6 demonstrate that for the two days over which we run the model instances, both (H) and (M) generally follow the same strategy. That is, for periods of constant load, the generators typically meet the load that solar cannot. For periods in which the load exhibits greater variability, the battery helps to peak shave the demand. Solar power is also used during these times to charge the battery for use later in the horizon such that the generator only operates at its “efficient level,” typically, at or above 90% of its rated capacity, or not at all. When solar power is being stored in the battery, under both the solutions from (H) and (M), one sees that the solar and battery power “mirror” each others’ profiles, even if said mirroring is offset by demand that must be met by solar (that is not met with the generator). The most pronounced qualitative difference between the dispatch strategies produced by (H) and (M) is the level of detail. While the former strategy is, by design, constant for each 60-minutes interval, that of (M) closely follows the dispatch strategy. This is particularly true of the battery and solar dispatch, while in both cases, the generator dispatch appears to be relatively constant. Quantitatively, fuel use in Phase I is 4.30 and 10.72 gallons for days 7 and 10, respectively. The corresponding fuel use in Phase II is 4.03 and 7.76 gallons. This disparity can be explained by the difference in battery use that occurs because of the added constraint to the minute model, i.e., that the battery must be charged to an 80% state-of-charge at the end of each day. This leads to increased battery use and greater fuel savings, generally, in the minute-level model.

One marked difference between our two test runs is that while the Day 7 case (Fig. 5) shows hour- and minute-level dispatch to follow each other less closely, the Day 10 case (Fig. 6) depicts the two strategies matching each other more closely. Day 7 possesses a more constant demand in the early part of the day, allowing the batteries in both cases to essentially meet the load, after which a highly variable demand is met from solar power while the battery is being charged with the excess energy produced. Because the demand is only registered at the hourly level for (H), the model is less agile in its response to meeting said demand. Towards the latter part of the day as the solar power diminishes, it appears that the battery in both solutions powers demand until that technology no longer suffices, in which case the generator turns on. For the Day 10 case, the demand is more variable at the beginning of the day. While (H) chooses to use the generator and some battery power to meet demand, (M) meets the load entirely with the battery. In (M), generator ramp rates and times are considered, and, hence, the more detailed model is limited by the reality of the generator’s performance to a greater extent. The dispatch strategies for both models match more closely in the middle of Day 10 than for Day 7. In the latter part of the day, the battery in the solution from (H) is overly optimistic about its performance capabilities (recall, the concentration gradient is lacking). The more realistic solution from (M) uses the battery less and therefore relies on the generator because solar power has diminished.

In addition, we conduct a parametric study that tests the effects of changing four inputs, specifically, (i) \( k_b^{\text{eff}} \) and (ii) \( k_a^{\text{eff}} \), the constant for each battery \( b \) associated with the temperature dependence of diffusion coefficients at the cathode and anode particles, respectively; (iii) \( \delta' \), the cost of fuel in time period \( t \); and (iv) \( \epsilon \), the overage load coefficient. The rationale for these choices is as follows: the battery constants are rougher estimates based on empirical testing than the others used in the model because they are a function of activation energies, whose values vary considerably as given in the literature. By contrast, some battery parameters such as \( k_b^{\text{eff}} \) and \( k_a^{\text{eff}} \) can actually be obtained from manufacturers and/or have been well established in the literature, and, as such, are much more certain. A realistic range for both of these less certain values lies between 6.054 and 24.216 K. The fuel cost is the subject of much debate, as it pertains to a “fully burdened cost of fuel,”
not simply to the value of the commodity in a civilian environment. The extra penalty results from the danger associated with transporting the fuel through a war zone, and is particularly subjective depending on the weight placed on this danger and the risk associated with the loss of human life. A realistic range lies between $25 and $100 per gallon. Finally, we choose values of $k$ between 0 and 0.05 to represent the cable losses in the system, which do not exceed 5% for lines under 300'.

We use the Latin hypercube sampling function within Matlab [38] to generate 1,000 random combinations of the four parameter values in question, and run the minute-level optimization model (M) for each of these using, without loss of generality, Day 7 demand; Matlab’s rstool runs require a few seconds on a standard desktop PC, while solving (M) requires between 30 s and 15 minutes per run, with an average run time of approximately two minutes (using the software given above). For each run, we record two responses of interest: the objective function value of (M) and the total fuel consumption using a quadratic fit, which works best for our tests. The upper panel in Fig. 7 shows the fit and 95% confidence interval bands for each of the original values of the four inputs we change, while the lower panel shows the same for a set of four arbitrary values. The dashed (blue) vertical lines correspond to the values of each input parameter; the dotted (purple) horizontal lines provide the values for the responses (objective function value and fuel consumption) for the given set of four inputs. Each fitted curve and confidence interval is based on the 1,000 runs we make. Generally speaking, fuel consumption appears to be relatively constant for the ranges of the parameters we vary with the exception of the overage load coefficient. The overall objective function and fuel consumption values are sensitive to both the fuel cost and the load coefficient.

Figs. 8 and 9 show a comparison between the modeled and fitted data for each response, i.e., objective function value and fuel consumption, respectively. The $R^2$ values close to 1 signify that the responses given in Fig. 7 closely match the objective function values obtained from running (M) with the prescribed set of four input values used in the parametric study (keeping all other parameter values for the Day 7 run the same).

6. Conclusions

We present two models for power dispatch in a microgrid - one, (H), a modification from an existing model which produces both a design and dispatch strategy based on hour fidelity, and the second, (M), which takes the dispatch strategy determined by (H) as given, and yields a more detailed, minute-level dispatch strategy from a model that incorporates the battery characteristic of a concentration gradient and the generator attributes of ramping and minimum up- and down-times. We show that while (H) can produce a credible dispatch...
strategy, \( (\hat{M}) \) is far better designed to handle the minute-level fluctuations in load. Quantitatively, fuel use decreases from the dispatch solution provided by \( (\hat{H}) \) to that provided by \( (\hat{M}) \) by about 6.6% and 38.1%, respectively, for the Day 7 and Day 10 case studies owing to increased battery use. On the other hand, instances of \( (\hat{M}) \), with current state-of-the-art hardware and software, would not be tractable for a horizon sufficiently long to produce an informed procurement strategy. Additionally, because demand for each minute throughout the course of a year cannot be accurately forecasted, by its nature, the dispatch should be solved for shorter time horizons. By using both models in these different manners, we combine their best attributes.

Practically speaking, the process described in this paper improves upon the work of Scioletti et al. [1] by providing a dispatch strategy...
that includes considerably more detail; specifically, (i) the load values are given at the minute, rather than the hour, level; (ii) the generators are associated with minimum up- and down-times, as well as ramp-up and -down times, not present in the Scioletti et al. [1] work; and (iii) the battery behavior incorporates a concentration gradient. These improvements allow the military to better compute an optimal, real-time dispatch strategy, as opposed to a procurement strategy yielded by Scioletti et al. [1].

Future work entails collaboration with researchers who run microcontrollers to compare our minute-level dispatch against theirs, which is, at the time of this writing, produced via heuristics. While said heuristics may be able to account for real-time control, they might also benefit from an optimized strategy to inform their decisions. This could aid Soldiers out in the field, as well as occupants of other remote sites such as those in sparsely populated rural areas, or industries that have off-grid operations, such as open-pit and underground mines in some countries.

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References